Notes: Introduction to AP Physics

Level 1: Class Business

The Three Rules of Physics
You are going to learn lots of rules in this class—laws that govern the heat in your teacup, the color of your eyes and the death of the universe. If these things sound hard, don’t panic—learning some of this will be easy. Learning other parts can be very hard, I won’t lie to you about that.

Perseverance is the most important thing you can take from this class. There will be moments in here when you are sure that you are stupid. There will be moments when you walk down the hall smiling because you know something about reality that very few people in this world understand. The dumbest I ever felt was in physics. The smartest I ever felt was in physics. The trick is not to let the emotions stop you. Don’t give up on yourself when you struggle. Don’t get cocky and stop learning when you doing well.

I’m going to give you the three rules of physics. You don’t have to believe them, but I will bring them up a lot.

The Three Rules of Physics

1. **Don’t Panic.** This is the first rule of physics. There will be a lot of moments when you will read a problem on a test and feel completely lost. Sometimes you genuinely are lost—other times, you are just momentarily disoriented, and if you take a deep breath and relax you’ll be able to work through it. Physics will often daze you, but that does not mean it will defeat you.

2. **If you can’t solve the whole thing, solve anything you can.** There are going to be lots of times when you will have no idea how to get to the answer. The best way to deal with this is to try to find something—anything. Can’t find the work? At least find the mass. You should write out the variables you know, draw a picture, solve for something easy that you can figure out... the important thing is not to throw your hands up and give up. The AP test gives enormous amounts of partial credit—the answer is generally only worth a point out of five or six.

3. **Don’t give up.** Physics belongs to those who are willing to bang their head against the wall of ignorance enough times to knock it down. This is not a race, though it will sometimes feel like it. This is not a competition, though I will sometimes use your drive to compete to motivate you. Perseverance, self-reliance and hard work are the keys to this game. Don’t be afraid to nerd out.

Tips for Doing Well
You do not need to be a genius to do well in this class. You will need to work hard (for some of you, harder than you’ve ever worked) but you don’t need to become a monk, forsaking all worldly pleasures and thinking only about physics. If you want to do well in this course, the best thing you can do for yourself is establish a few good habits. Good habits are the key to success in this class.

1. **DO. NOT. CRAM.** Do I have students who cram for tests and do well on them? Yes. Do these students ever do well on the actual AP test? No. Even if you are a mediocre student, if you are diligent and review regularly, you will do significantly better than a higher level student who crams. This course is known
for covering an epic amount of information. Plus, physics builds on itself. You can’t forget a section--it will appear again and again, haunting you when you least expect it on tests and quizzes.

2. **Make every effort to solve example problems and practice problems before you look at the solutions.** Physics is not a spectator sport. You must be actively engaged in it, from the first time you see the information to looking over your mistakes after a test. If you just passively read the solutions, I guarantee you that you will do poorly.

3. **Talk to others.** This goes for both the strong students and the weak students. If you are a weak student, you may be embarrassed about asking questions. Don’t worry--physics eventually makes us all feel (at times) stupid. Physics will someday (soon) make the smartest kid in this class feel like a moron. So ask for help--not just from me but from your classmates. If you are a strong student, you may feel like explaining problems to others is a waste of time, but let me reassure you that this will make you extremely competitive on the test. The AP Physics exam really favors those who can clearly explain their thinking to others--not those who can just find the answer. Every single one of the students who got 5s on the AP test were very involved in explaining physics to other students.

4. **Look over your mistakes.** This is the most painful. Most students will probably have at least one low test or quiz in here. Don’t panic about your grade. Nearly all hard workers maintain solid As and Bs in here. If you struggle, I will help you out. There is usually a very generous curve that keeps your grade up, but it still hurts to see your paper covered in pen. In this class, even perfect papers are usually covered in pen, either because I want to point out exactly what amazing things you did, or I want to point out areas where you may lose points in the future but didn’t lose points on this particular question. Look over every comment I make. If I took the time to write it, it is important. Try the problems again at home. Take notes on what type of mistakes you made.

5. **Get help.** If you’ve ever walked past my classroom after school, chances are you saw me sitting with a small group of students. Those were AP students who were confused by a reading or couldn’t solve a couple homework problems. I’m kinda obsessed with my job and I care deeply about my students. I encourage you to exploit this fact to get as much help as possible from me whenever you can. Class time is fine, but you’ll have to share me with a huge number of other students. After school, we can often offer you much more one-on-one time.

**Website**

You will be constantly visiting the class website. You will want to bookmark it on your phone and/or your laptop. The notes and practice pages and other resources will be posted online. Most students prefer to read the notes online, but if you do want to have a hard copy of the notes, you will need to print them out at home. I know this is rough but it would take an inconceivable amount of paper to provide all students with paper copies. The notes are just too big. Here is the website:

[storyphysics.weebly.com](http://storyphysics.weebly.com)

**Email**

You will probably need to email me at some point during this class. I may also need to ask you questions about your grade/assignments. **At this time, I want you to send me a brief email.**

**My email address is** [storystelznerb@hopewellarea.org](mailto:storystelznerb@hopewellarea.org)

**On the subject line, put your full name, followed by the word “email”.** For example, John Smith Email. Please don’t write anything snarky or inappropriate in the body. I won’t contact you unless I need to.

**Remind**

You may have already used this app with your coaches and other teachers. It allows me to send you texts about important class information. I will not be able to see your number or to text you individually. You won’t be able to text me at all and you won’t have my number. I won’t use this too often, but it can be helpful. For example, if we have a snow day when we were supposed to have an exam, I might text you and tell you the test is moved so that you won’t stay up studying. Or I may remind you on Sunday about a major test if it happens to fall on a Monday. I will not fill your inbox with pictures of puppies or fascinating science articles. I will use this pretty infrequently, but I often use it to convey vital information that will make your life harder if you miss it.
Follow the directions below:

Quizlet

Quizlet is a widely used app, both at the high school level and (even more so) at the college level. It basically allows you to create flash cards, which you can then use to quiz yourself. The good thing is, because so many people are using this app, there are already flashcards made for any topic you can imagine. Feel free to use this in other classes as well. In this class, there isn't a ton of memorization. When there is, I will try to provide you with a quizlet to make it easier.

For now, you will need to make an account and sign up for this class.

1. Go to http://quizlet.com/
2. Sign up and create an account.
3. Once you are in, a toolbar will appear on the left. Click on “+Join or create a class”
4. You can search for our school

Now let's actually learn stuff.

Prefixes

You need to memorize these. Now. Sorry! On the Welcome to AP Physics Practice Page, I've put a link to a quizlet that will help you learn these quickly. You can even play games.
**Level 2: Graphs and Data Analysis**

**Level Goals:**

1. Make appropriate graphs
2. Use the graphs to create equations that relate two variables.

\[ F = ma, \ V = IR, \ W = Fd, \ v_f^2 = v_0^2 + 2ad \ldots \] where did these physics beauties come from? Some of them were derived using mathematics, but others were made using data and experimentation. How does data give birth to equations? How can we use what we find in a laboratory to create statements that predict reality all over the world (and reality on other worlds)?

You need to understand that data leads to graphs, and most graphs exist for no other purpose that to illuminate the relationship between two variables. In this section, we are first going to review graphing expectations and the basic information that graphs provide us with. Then we will explore how to create and use graphs that can help us create equations.

**The Very Basics (skip this if it looks to easy)**

You should be familiar with graph construction (by hand and on a calculator). This is a topic that often appears on AP exams and is an easy way to score points on any assignment.

**Note:** When you are told to graph Apples vs. Oranges, the first thing goes on the y-axis. The second thing is on the x-axis.

Fill in the following table and plot the points on the grid below as distance versus time. Be sure to correctly label the graph (axes labels, including units, and title)
**Draw the best fit line through your data points.** Your best fit line is not a connect-the-dots line. Nor should it connect the first and the last point on your graph. What you should do is lay your ruler down and position it so it is as close as possible to all the points. The run your pencil along the line of the graph. It should look something like this:

**Finding the Slope**

You should remember how to calculate slope, but here is a brief review.
Pick two points (X-value, Y-value) that are on the line. The easiest points to pick are points where the line crosses the grid of the graph paper so you don't have to extrapolate the x and y values of the points. In this example, Point 1 could be (5,10), i.e., \( X_1 = 5 \) and \( Y_1 = 10 \), and Point 2 could be (10,20), i.e., \( X_2 = 10 \) and \( Y_2 = 20 \).

Plug these two points into the following formula to calculate the slope:

\[
Slope = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(20 - 10)cm}{(10 - 5)sec} = \frac{10cm}{5sec} = 2\frac{cm}{sec}
\]

**Important Point**

If you are using a best fit line, choose two points on the line that are easy to read. **Do not use any of the points on your given data table.** Why? Because your data could be unreliable. It doesn't matter how careful you are—you are human and your equipment is human. It is not always a perfect mirror of reality. You should therefore never trust one data point for anything. Always use the trend line you created. Pick two points on it and move forward. If you don't do this, the AP grading process requires that I mark you down every time.

**Types of Graphs**

**Horizontal Graphs**

Occasionally data will produce a graph with an appearance similar to the diagram to the right. This graph indicates that as the variable plotted on the x-axis changes, the variable on the y-axis stays the same. There is no relationship between the two variables. When asked to state what this type of graph indicates, one says, “y is independent of x.” This means that x does not influence y. Sort of the way Justin Bieber has no influence on good music.

The line has a slope of zero and the intercept is the same value as every other point on the line. Therefore the algebraic relationship is \( y = b \), where b is a number.

**Directly Proportional Relationships**

We say that the variable on the y-axis is **directly proportional** the variable on the x-axis when you get graphs like the ones below. That phrase “directly proportional” tells you that if the variables doubles, so does the other. If you triple one, the other triples. You can tell if the relationship between two graphed variables is directly proportional because they always look like this:
Notice the trend? Lines. Yup, straight lines. All of them pass through the origin (0.0).

These guys are easy. They always follow the same pattern. The equation for these lines is always:

\[ y = mx \]

where \( m \) is the slope.

This is the equation for our data. We fit it into \( y = mx \)

Practice Problem: Finding the Equation

Graph the data below and write the equation that explains the relationship between the two variables.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Position (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>6.1</td>
</tr>
<tr>
<td>2</td>
<td>9.9</td>
</tr>
<tr>
<td>3</td>
<td>14.5</td>
</tr>
</tbody>
</table>

**Solution**

\[ y = 4.73x \]
Linear Graphs
All straight line graphs are technically **linear graphs**. The directly proportional graphs we looked at above are a type of linear graphs. Not all linear graphs pass through the origin. However, when our line of best fit does not pass through the origin, the graph and equation get a little more complicated.

Notice these are still straight lines, but they no longer pass through the origin. If we want to find the equation for this graph, we need to know the slope and the y-intercept. This is where the line crosses the y axis.

Note: The y-intercept does not always need to be positive.

Once, you have the y intercept, you still need to find the slope, just like you normally would.

After you find the slope, you can create the equation between the two variables. The example below shoes this. The equation for a linear relationship is \( y = mx + b \).
Graphing lines using $y = mx + b$

Next use the slope to find a second point on the line.

First plot $b$, this is the $y$-intercept (one point on line).

Graph $y = \frac{2}{3}x + 2$

Point A

Use slope to get point B.

Line $y = 2 \frac{1}{3}x + 2$
Practice Problem: Finding the Equation for a Linear Relationship

Sketch the graph below on graph paper. Draw the best fit line and use it to find the equation that describes the relationship between total fat and total calories.

<table>
<thead>
<tr>
<th>Sandwich</th>
<th>Total Fat (g)</th>
<th>Total Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamburger</td>
<td>9</td>
<td>260</td>
</tr>
<tr>
<td>Cheeseburger</td>
<td>13</td>
<td>320</td>
</tr>
<tr>
<td>Quarter Pounder</td>
<td>21</td>
<td>420</td>
</tr>
<tr>
<td>Quarter Pounder with Cheese</td>
<td>30</td>
<td>530</td>
</tr>
<tr>
<td>Big Mac</td>
<td>31</td>
<td>560</td>
</tr>
<tr>
<td>Arch Sandwich Special</td>
<td>31</td>
<td>550</td>
</tr>
<tr>
<td>Arch Special with Bacon</td>
<td>34</td>
<td>590</td>
</tr>
<tr>
<td>Crispy Chicken</td>
<td>25</td>
<td>500</td>
</tr>
<tr>
<td>Fish Fillet</td>
<td>28</td>
<td>560</td>
</tr>
<tr>
<td>Grilled Chicken</td>
<td>20</td>
<td>440</td>
</tr>
<tr>
<td>Grilled Chicken Light</td>
<td>5</td>
<td>300</td>
</tr>
</tbody>
</table>

Solution

\[ y = 12.9x - 144 \]

Here is a [site](#) where you can see the solution in more detail.

Curved Graphs

Not all graphs are straight lines. In fact, a lot of relationships that we will encounter in this class are non-linear. No matter what, when you get a data set (like the one above) you are going to need to graph it. Once it is graphed, the first step to figuring out its equation is to recognize its shape. In this class, you will need to recognize parabolas and inverse relationship graphs at a glance.

Parabolas

If you graph data with a parabolic relationship, it will should look something like the graph on the right. In this class, the graph will generally go through the origin when it is parabolic.

When you plot data like this and you need to draw a best fit line, try to create a curve that smoothly follows the path of your data points and has a shape similar to the one on the right. **The equation for a quadratic equation is** \[ y = mx^2 \].

How do we figure out the m (slope) for that equation, though? How can we possibly calculate the line when it is curved? We could use calculus, but this is an algebra based course, so let's talk about ways around that for now. In order to pull this off, we need to learn a new and incredibly useful skill called **linearization**. This means that we take a curved graph and plot it in such a way that it becomes a graph with a straight line.

Here’s how you do this:

1. Plot the data points.
2. Draw a best fit line.
3. Figure out the shape (horizontal, directly proportional, linear, parabola...)
4. Write down the general equation for that type of line. In this case:

\[ y = mx^2 \]
5. Here is where it gets funky. Look at the y, look at the x. See how the x is squared? Create a new data table that plots y vs. x².

6. Graph the new relationship, in this case y on the y axis, x² on the x axis.

If that was hard to follow, it is okay. I’ll show you how.

**Example Problem: Finding the equation of a Curve**

Use the data below to create an equation that relates time and displacement.

<table>
<thead>
<tr>
<th>time (s)</th>
<th>displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>63</td>
</tr>
<tr>
<td>10</td>
<td>98</td>
</tr>
</tbody>
</table>

1. Plot the data points.

2. Draw a best fit line.
3. Figure out the shape (horizontal, directly proportional, linear, parabola...)
   In this case, that sure looks like a parabola to me. That means the equation I’m looking for will have the format \( y = mx^2 \). Now, I need to find \( m \).

4. Create a new data table that tells you the values that match the format of the equation.

   \[
   \begin{align*}
   y &= mx^2 \\
   d &= mt^2
   \end{align*}
   \]

   Notice that the equation suggest time will be squared. So, we figure out what these values are:

<table>
<thead>
<tr>
<th>time</th>
<th>time^2</th>
<th>displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td>63</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>98</td>
</tr>
</tbody>
</table>

   Notice how the middle column shows the value of time (on the x axis) squared. I chose to do that because the equation I ultimately think I’m looking for looks like this:

   \[ y = mx^2 \]

5. Now, plot \( y \) values (displacement) vs. the new x values (time^2). Make sure you adjust your units. Once you’ve done that, add a best fit.

   \[
   \begin{array}{cc}
   t^2 & y \\
   \hline
   0 & 0 \\
   1 & 3 \\
   9 & 8 \\
   16 & 17 \\
   36 & 35 \\
   64 & 63 \\
   100 & 98 \\
   \end{array}
   \]
Ta-da! You just turned a difficult-to-work-with curved graph into a nice straight line. This is called **linearization**. Now, you can measure the slope to find $m$. Do that now.

6. Find the slope (just like you normally would).
   If you did this right, you should get a slope of about 0.97 or 1.

7. Plug the values into the equation.

   \[ y = mx^2 \]
   
   or, if your slope was 1, it's just:

   \[ \text{displacement} = (0.97) \text{time}^2 \]

**Practice Problem: Create an Equation From a Curve**

<table>
<thead>
<tr>
<th>position (m)</th>
<th>Work (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.25</td>
<td>0.32</td>
</tr>
<tr>
<td>0.5</td>
<td>1.3</td>
</tr>
<tr>
<td>0.75</td>
<td>2.7</td>
</tr>
<tr>
<td>1</td>
<td>5.1</td>
</tr>
<tr>
<td>1.25</td>
<td>7.8</td>
</tr>
<tr>
<td>1.5</td>
<td>11.3</td>
</tr>
</tbody>
</table>

Using the data below, create an equation that relates position and work. Then, find out how much work would need to be done to move the object to position 3 m.

**Answer**

If you solved this problem correctly, you should get the equation \(\text{Work} = 5 \text{position}^2\). If we just plug and chug the numbers into this equation, we find that 45 J of work need to be done to move the object to 3 m.

**Inverse Relationship**

While parabolas are by far the most common type of curved graph you will encounter in AP Physics, we do sometimes see graphs that represent inverse relationships. In this class, as $x$ increases, $y$ decreases. They look like this:
This graph means the variables are related using the following equation:

\[ y = \frac{m}{x} \]

Think about linearization. If you plot your data and realize that the graph shows an inverse relationship, how do you think you should go about linearizing it? For a parabolic graph, you plotted \( y \) vs. \( x^2 \). For the inverse graph, what do you think you would plot on the x axis to linearize it?

If you said \( 1/x \), you are cooler than Katy Perry. She wishes she had your mad math skills.

That's right, to linearize a graph that shows an inverse relationship, you would plot \( 1/x \). Then you proceed the same way you did in the last problem, by finding the slope and plugging this info into the equation.

**Practice Problem**

Find the equation that relates volume and pressure using the data table below.

<table>
<thead>
<tr>
<th>Pressure (kPa)</th>
<th>Volume (cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>31</td>
</tr>
<tr>
<td>180</td>
<td>34</td>
</tr>
<tr>
<td>140</td>
<td>44</td>
</tr>
<tr>
<td>100</td>
<td>62</td>
</tr>
<tr>
<td>85</td>
<td>73</td>
</tr>
<tr>
<td>70</td>
<td>88</td>
</tr>
<tr>
<td>60</td>
<td>103</td>
</tr>
</tbody>
</table>

**Solution**

If you did your linearization correctly, you would have ended up plotting the following data:

<table>
<thead>
<tr>
<th>1/Pressure (1/kPa)</th>
<th>Volume (cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>31</td>
</tr>
<tr>
<td>0.005555556</td>
<td>34</td>
</tr>
<tr>
<td>0.007142857</td>
<td>44</td>
</tr>
<tr>
<td>0.01</td>
<td>62</td>
</tr>
<tr>
<td>0.011764706</td>
<td>73</td>
</tr>
</tbody>
</table>
And creating the following graph:

![Graph of Volume vs. 1/P](image)

Then calculate a slope of 6185 cm³ kPa

Which leads you to the equation:

\[ V = \frac{6185}{P} \]

**To Summarize**

In case you are curious, linearization can be done with any graph shape, so long as you are able to guess at the form the equation will ultimately take. The four outlined above are the ones that appear 99% of the time on the AP test, and will do the job for this class. I’ve made you a chart to help summarize these below.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Looks Like</th>
<th>Equation</th>
<th>To linearize</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td><img src="image" alt="Linear Graph" /></td>
<td>( y = mx + b )</td>
<td>No need. Just use slope equation ( \text{Slope} = \frac{\Delta y}{\Delta x} )</td>
</tr>
<tr>
<td>Parabola</td>
<td>$y = mx^2$</td>
<td>To linearize plot $y$ vs. $x^2$</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
<td>-------------------------------</td>
<td></td>
</tr>
<tr>
<td><img src="https://example.com/parabola_graph.png" alt="Parabola Graph" /></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverse Relationship</th>
<th>$y = \frac{m}{x}$</th>
<th>To linearize plot $y$ vs. $\frac{1}{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="https://example.com/inverse_graph.png" alt="Inverse Relationship Graph" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Level 3: Graphing in Excel**

In the last section, our discussion of graphing dealt mainly with drawing graphs by hand. This is very important, because on the AP test and on tests in here, you will not have access to computers. However, whenever we do labs in here (which will be pretty often), you are allowed to use Excel, a program that allows you to make and analyze graphs easily. Most of you have used Excel before. If you haven’t, someone in your group probably has and can help you out.

**Watch this Intro to Excel:** This reviews how to enter data into Excel. If you are already very familiar with Excel, you can skip to 1:30. You do need to know how to add a trend line and how to find the $R^2$ value.

[https://www.youtube.com/watch?v=IT4n_INCONM](https://www.youtube.com/watch?v=IT4n_INCONM)
One thing that is incorrect about the video— in this class, we will not always have linear graphs (as discussed above). When creating a trend line in the graph, you are offered lots of options (see the screen capture on the right) polynomial, inverse, linear...

For the purposes of this course, you will most often use linear and Polynomial.

**Key Ideas**

- If you think your graph is quadratic, click “Polynomial” and then make sure the order is “2”.
- If you are dealing with an inverse relationship graph, choose “Power”.

How do you know what the graph shape is? Inverse relationship graphs are pretty easy to spot, but it can sometimes be difficult to tell whether a graph is linear or quadratic, depending on how bad your data is. How do we figure out what trend line best fits our data? This is where the R² value becomes useful.

Look at the shape of your data points, and take your best pick of which trend line to pick. Then- before you click “OK” check the box at the very bottom labeled “Display R-squared value on chart”.

The R-squared value will tell us how well our data matches that particular trend line. **Values that are close to 1 are really good.** Aim for 0.95 and above. R-squared values that are lower suggest that your data does not match that trend line very well. In that case, you should either try out a different trend line or go back and look at your data again.

**Level 4: Problem Solving Expectations**

The AP Physics test (and all of my tests) will offer enormous partial credit. In order to earn that partial credit, however, you need to do all the following, every time you solve a problem. Every. Single. Time.

1. List your variables.
2. **Write the correct equation without numbers in it.**
3. Rearrange the equation to solve for the variable you are curious about.
4. **Plug in the numbers on paper (before you put anything in your calculator)**
5. Write your answer with correct units.
6. Box up your answer (It's physics Christmas!)

I want you to follow the list above on every problem you possibly can. The ones that are bolded are the steps that frequently earn you partial credit on the AP tests (and my tests). The others will just keep you from getting confused and will make your life easier.

**Easy Easy Practice Problem**

A boy, who is originally at rest and has a mass of 44 kg, jumps off a snowboard (10 kg) with a velocity of 5 m/s. What is the velocity of the snowboard after he jumped?

*I know that you may be able to solve this with numbers in a jiffy. But from now on, you will first need to solve for it in algebra form. For this problem, let’s say M1 is the boy and M2 is the skateboard.*

\[ p = p' \]
\[
\begin{align*}
m_1 v_1 + m_2 v_2 &= m_1 v_1' + m_2 v_2' \\
0 &= m_1 v_1' + m_2 v_2' \\
-m_1 v_1' &= m_2 v_2' \\
\frac{-m_1 v_1'}{m_2} &= v_2'
\end{align*}
\]

Only now - after I've gotten it to this step, can I plug the numbers in.

\[
\frac{-44}{10} = -22 \text{ m/s}
\]

If you did not practice this in your last physics class, it feels incredibly time consuming and annoying. You will hate it at first. I’m going to reassure you right now that it is not just me who makes you do this - this is a widely expected practice in physics and engineering. Why do we do this? Several reasons:

1. It will make you an absolute Hulk at algebra.
2. Sometimes (often) the AP Physics test will give you a problem without numbers. You heard me - they will want you to solve the whole problem without touching numbers. Here is an example (one of many) from an actual AP exam:

![Diagram of a bullet fired at a block]

1976B2. A bullet of mass \( m \) and velocity \( v_0 \) is fired toward a block of mass \( 4m \). The block is initially at rest on a frictionless horizontal surface. The bullet penetrates the block and emerges with a velocity of \( \frac{v_0}{3} \).

(a) Determine the final speed of the block.
(b) Determine the loss in kinetic energy of the bullet.
(c) Determine the gain in the kinetic energy of the block.

... and here is what the answers look like:


a) Apply momentum conservation. \( p_{\text{before}} = p_{\text{after}} \) \( mv_0 = (m)(v_0/3) + (4m)(v_F) \) \( v_F = v_0/6 \)

b) \( KE_f - KE_i = \frac{1}{2} mv_0^2 - \frac{1}{2} m (v_0/3)^2 = 4/9 mv_0^2 \)

c) \( KE = \frac{1}{2} (4m)(v_0/6)^2 = 1/18 mv_0^2 \)

Don't worry if this looks scary. We aren't there yet. We will get there.

3. **It makes it so much easier to solve the problem with different values.** Let's say I now asked you how fast the skateboard from the previous problem would go if a girl with a mass of 33 kg jumped off with a velocity of 3.5 m/s. If you solved with numbers, you would need to start the problem all over again. If you solve with variables, you can just plug in the new numbers and solve in a flash.

\[
\frac{-m_1 v_1'}{m_2} = v_2'
\]

\[
\frac{-33(3.5)}{10} = -11.5 \text{ m/s}
\]
4. If you keep practicing physics and engineering, numbers will become annoying to you. This is strange. When you first start doing this, it absolutely sucks. You hate it. It is like running your first lap or lifting your first weight - it is agony. But then you start to get used to it... and after a while, you really do start to prefer it. It takes a long time...my AP students hated this last fall, but by the end of the year most of them don't notice whether the problem gave them numbers or not. They just solve.

Level 5: Problems Without Numbers

Video Review

Before we move on, I'm going to have Ms. Twu review a few key ideas from previous science and math courses that you will need to know throughout this course. We will frequently watch other instructors teach or do demonstrations in this course. I do this for a few reasons:

1. I want you to get used to different instructors. You will have instructors from all over the world when you go to college. Many will teach in ways you have never seen before. Some will have accents that are difficult for you to understand at first. You need to get comfortable learning from a wide range of teachers.

2. You can pause the videos. You can't pause a lecture when the instructor is confusing. This way you can stop what you are doing and talk to your group members, see if they understood. You can go back and listen again. You can watch them at home until you understand them. You can pause and take notes.

3. Time, both yours and mine. As you can imagine, these notes take a long time to make. By the time you finish this class, you will have a not-so-small book from me. Not everything can be efficiently explained on paper, however. Sometimes you need to watch other people show you how it's done. Lecture can be very slow (though I will do it). This way, I can show you clips of exactly what you need to know.

Ms. Twu is a great teacher and you could, technically, go home and learn this entire course by watching her videos.

Review with Twu: AP Physics 1: Introduction 1: Standard Units & Unit Dimensional Analysis

https://www.youtube.com/watch?v=pN3Zizzy3RA#t=252

Ms. Twu does miss one volume equation you need to know, the volume of a sphere:

\[ V_{\text{sphere}} = \frac{4\pi r^3}{3} \]

MEMORIZE THESE EQUATIONS if you haven’t already

Solving Problems without Numbers

As I mentioned before, you will sometimes be asked to solve problems without numbers. How do we do this? We pretend that the letters given to us are numbers. Let me show you how this is done in a practice problem.

Practice Problem: Solving Without Numbers

A car at rest accelerates uniformly at a rate \( a_1 \) to a velocity \( v_1 \). How far does the car go during this acceleration, in terms of \( a_1 \) and \( v_1 \)?

Solution

\( a = a_1 \)
\( v_0 = 0 \)
\( v_f = v_1 \)

\[ v_f^2 = v_0^2 + 2ad \]
\[ \frac{v_f^2 - v_0^2}{2a} = d \]
Now, plug in the letter exactly the way you would plug in the numbers.

\[
\frac{v_1^2 - 0}{2a_1} = d \\
\frac{v_1^2}{2a_1} = d
\]

And, believe it or not, that’s your answer. You’re done. Box it up and call it a day.

Not so bad, huh? They really aren’t that scary. Now, that said, this was an easy problem. Sometimes at the last step, you will need to simplify or cancel stuff out. Sometimes you will need to take this answer and apply it to a future problem. Let’s try one of those.

Practice Problem: Continuing without Numbers
While the car in the previous problem underwent the acceleration described, its engine did work equal to \(3W_0\). How much force did the engine apply to the car?

Solution
\(W = 3W_0\)
And, from the previous problem:
\(d = \frac{v_1^2}{2a_1}\)
Remember that work equals force times distance:
\(W = Fd\)
\(F = \frac{W}{d}\)
Now, plug in the letters as though they were numbers:
\(F = \frac{3W_0}{\left(\frac{v_1^2}{2a_1}\right)}\)
…and we use a little algebra to make it prettier by flipping that lower denominator up:
\(F = \frac{3W_0(2a_1)}{v_1^2}\)
\(F = \frac{6W_0a_1}{v_1^2}\)
Box it up, that is your answer. This is very typical of the type of work you might be asked to do.

Proportional Reasoning
Sometimes, AP Physics problems will ask you to compare to situations without numbers. In that situation, you will need to use something called **Proportional Reasoning**. This type of mathematical thinking helps us figure out how, if we change one variable in a problem, the rest of the variables might be effected.

It comes up a lot. Let’s talk about a super easy equation.

\(F = ma\)

Now, what if I asked you a question like this:

*A cart with a mass of \(m_0\) is pushed forward with a force \(F_0\) and experiences an acceleration of \(a_0\). Later, a student adds a weight to the cart, making the mass \(2m_0\). She then pushes the cart with the same force. How does the new acceleration, \(a_1\), compare to the original acceleration, \(a_0\)?*

If reading that problem made you feel like this....
...don't worry, that's normal. I have trouble understanding that problem, and I'm the one who wrote it. These things will always make you feel completely disoriented. Don't worry, it happens to almost everyone.

The trick is not to panic and to write the equation down.

\[ F = ma \]

The final question are asks you about acceleration so you will need to solve for that.

\[ a = \frac{F}{m} \]

At this point, we are going to set up a proportion between the two variables that are changing. In this case, the mass and the acceleration are changing.

To set up a proportional relationship

1. Write the equation:

\[ a = \frac{F}{m} \]

2. Get rid of factors that don't change and replace the equal sign with a proportion symbol.

\[ a \propto \frac{1}{m} \]

Notice we got rid of force, because the problem specifically says the force does not change. That little fish symbol means “proportional to”. It tells us that if we change the acceleration, m will have to change, too, and the amount one changes effects the amount the other changes.

3. Write down how much the variable will change. Do not use an = sign.

In the problem, they tell us m doubles. So we write:

\[ m \rightarrow 2 \]

Notice that I didn’t use an equal sign, because the mass is not equal to two, it just doubled.

4. Plug this factor into the proportion.

If you plug this factor into the proportion, you can see exactly how much the acceleration changes.

\[ a \propto \frac{1}{2} \]

This means the new acceleration is half as large as the old acceleration. So you can say:

\[ \frac{a_0}{2} = a_1 \]

That might seem like a harder way of doing a problem you could have solved much more easily. Let's try this with a harder problem.

Practice Problem

The volume of a sphere is doubled. How does its new radius, R, compare to it's original radius, \( r_o \)?

Solution

\[ V = \frac{4}{3} \pi r^3 \]

\[ V \propto r^3 \]

\( V \rightarrow 2 \)
\[ \sqrt[3]{2} \propto r \]
\[ \sqrt[3]{2} = 1.3 \]

That means the new radius is 1.3 times the old radius.