Notes: Hydrostatics

Level 1: Introduction to Fluids

Fluids are substances that can flow. We will deal with two main types of fluids: liquids and gases. Both of these obey the laws discussed in this chapter, but there some pretty obvious differences between them. In a liquid, the molecules are close together and often interact with one another. In gases, on the other hand, molecules tend to be far apart and fly around in wild patterns (which we will focus more on when we get to thermodynamics). Fluids can be poured into another container, while gases will just spread out until they fill their container. Neither liquids nor gases have a shape of their own.

This unit is divided into two parts: hydrostatics (which deals with fluids at rest) and hydrodynamics (which deals with fluids that are flowing and moving). In this first set of notes, we will only deal with hydrostatics.

Density

Here is an old riddle: which is heavier, a pound of feathers or a pound of lead weights? It seems easy on paper, but most people will go for the weights if you ask them. Why? Because they aren’t thinking about the weight of the feathers, they are thinking of their density. Lead is a very dense material - one pound of material probably fits easily in your hand. Feather are not very dense - you would probably need a giant bag to hold 1 pound of feathers.

The symbol we use to describe density is $\rho$, pronounced “rho”. The equation for density is really easy and obvious.

$$\rho = \frac{\text{mass}}{\text{volume}}$$

Mass is measured in kg and volume is measured in m$^3$, so the units of density are kg/m$^3$.

A Note On Volume

When asking you about a volume, the AP test can do a few things to mess with you. First, instead of straight up giving you the volume, they could give you the dimensions instead. For example, they could tell you a cylindrical tank has a height of 2 m and a radius of 60 cm. It is important to keep in mind three volume equations:

$$V_{\text{rectangular prism}} = \text{width} \times \text{length} \times \text{height}$$

$$V_{\text{cylinder}} = \pi r^2 h$$

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

If you don’t already have these memorized, you should. They come in handy in a wide range of classes and situations.

Dimensional Analysis with Volume

The standard units for volume is m$^3$ and many of your problems will give you volume with these units. Every now and then, however, the people who write the AP Physics test get truly evil. They will give you volume in different units.

For example, they could tell you that a tank of water has a 1000 cm$^3$. This is frustrating, because the standard units for volume are m$^3$. How do you convert? This isn’t as straight forward as a simple conversion. The cube means that you need to approach the conversion a little differently. There are lots of ways to do this. I’ll teach you the one that has been easiest for students in the past.

1. **Put parenthesis around the cm$^3$ (or whatever it is you are converting)**
   Sounds strange but stick with me- you’ll see what I’m doing in a minute.

   $1000 \ (cm)^3$
2. Write the conversion in numeric form

\[ 1000 \left(10^{-2}m\right)^3 \]

3. Distribute the cube to everything inside the parenthesis.

\[ 1000 \left(10^{-2}\right)^3 m^3 \]
\[ 1000 \left(10^{-6}\right) m^3 \]

4. Simplify.

\[ 1000 \left(10^{-6}\right) m^3 \]
\[ 0.001 m^3 \text{ or } 1 \times 10^{-3} m^3 \]

And that’s the conversion \(1000 \text{ cm}^3 = 0.001 \text{ m}^3\)

The Density of Water

Water has a very specific density at standard temperature that you need to memorize:

\[ \rho_{water} = 1000 \frac{kg}{m^3} = 1 \frac{kg}{L} \]

We don’t think of water as being very dense, but it is. Ask me to stop by your table, and I can show you exactly how compact 1000 kg of water is.

Specific Gravity

How can we predict if something will float in water or not? This is surprisingly easy. We can use the specific gravity equation:

\[ \text{specific gravity} = \frac{\text{density of an object or substance}}{\text{density of water}} \]

Specific gravity is a ratio that compares the density of an object (or substance) to the density of water. If the object is more dense than water, the numerator will be larger, the specific gravity will be greater than 1 and the object will sink. If the object or substance is less dense than water, the denominator will be larger, the specific gravity will be less than 1 and the object will float. If the object has the same density of water (rare but possible), the object will be suspended in the water- it won’t bob up to the surface and it won’t sink to the bottom.

The specific gravity is a characteristic of the object that depends upon what it is made of. In the chart below, you can see the specific gravity or different materials.

<table>
<thead>
<tr>
<th>substance</th>
<th>specific gravity</th>
<th>substance</th>
<th>specific gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline A</td>
<td>0.74</td>
<td>Chicken Fat</td>
<td>0.924</td>
</tr>
<tr>
<td>Petroleum Oil</td>
<td>0.82</td>
<td>Fish Oil</td>
<td>0.928</td>
</tr>
<tr>
<td>Hydraulic Fluid</td>
<td>0.86</td>
<td>Water</td>
<td>1</td>
</tr>
<tr>
<td>Butter</td>
<td>0.87</td>
<td>Freon 11</td>
<td>1.49</td>
</tr>
<tr>
<td>Lard</td>
<td>0.875</td>
<td>Mud</td>
<td>1.73</td>
</tr>
<tr>
<td>Olive Oil</td>
<td>0.91</td>
<td>Sand</td>
<td>1.92</td>
</tr>
<tr>
<td>Vegetable Oil</td>
<td>0.92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Practice Problem

You pour Freon 11, hydraulic fluid, water and vegetable oil into a jug. After everything settles out, the fluid appears layered. What would be on top? What is on bottom? List the materials from top to bottom.

Solution

Hydraulic fluid, vegetable oil, water, Freon 11
Take a look at the equation for specific gravity again. What do you think the units for specific gravity are?

If you said, “There would be no units,” then, congratulations, you’re a genius. If you don’t understand why this is, talk to your group members or Ms. Story to make sure you do.

Supplemental Video (not required) Going into the medical field? This short nursing school video explains how specific gravity can be used in urine analysis to help diagnose a wide range of problems.

https://www.youtube.com/watch?v=0PrXv4pdg-M

Water is Weird

Normally, when objects get hot, they expand and become less dense. This means their specific gravity goes down and they are more likely to float. On the other hand, when objects get cold, they typically shrink and become more dense-and more likely to sink when placed in water.

Okay, with that in mind, tell me why the picture below makes no sense:

Think about it: why does ice float? Shouldn’t it get denser as it cools, increase its specific gravity and sink? Yes, according to the general rule regarding most substances, it should sink. But water is weird in lots of ways- magical ways, really. It has lots of quirky little habits governed by chemistry we won’t get into here. Here is what you need to know:

As water cools from 100°C (boiling) to 4°C, it act normally- it get more dense, starts to sink, the whole shebang.

Between 4°C and 0°C (freezing), however, it starts to expand. Once it is ice- it expands even more. As it expands, its density decreases, its specific gravity goes down, and it floats.

The big idea here: Ice is less dense than water, so it floats. That is weird.

This is actually one of the reasons life on earth is able to survive- lakes and rivers will form ice on top of their surfaces rather than the bottom. This means that marine life can still exist below the ice. In fact, the ice acts as an insulator, keeping the water below the ice slightly warmer. Without this property (and other weird watery properties) life on earth could not exist. In fact, there are some of scientists who believe that life can’t exist without water- hence astronomers’ obsession with finding other planets with liquid water on them.

On the other hand, this is also why we have so many potholes. Water (in liquid form) seeps into tiny cracks in the road. Then the temperature drops, the water freezes and expands. It is a surprisingly strong substance, so the cement moves, cracks and crumbles. This is why so many new potholes appear in winter, especially a winter where temperatures fluctuate a great deal.
Practice Problem

Example 18.1 Turpentine has a specific gravity of 0.9. What is its density?

Solution

Solution. We can use the equation for specific gravity to find that

\[ \rho_{\text{substance}} = (\text{sp. gr.}) \rho_{\text{water}} \implies \rho_{\text{turpentine}} = (0.9)(1000 \frac{\text{kg}}{\text{m}^3}) = 900 \frac{\text{kg}}{\text{m}^3} \]

Pressure

Pressure is how much force is applied to a particular area. Here’s the equation:

\[ \text{Pressure} = \frac{\text{Force}}{\text{Area}} \]

This is often confused with force, which is different. Think of a heavy weight- nothing big enough to hurt you, but heavy. Imagine lying underneath it- you’d feel a bit squished, right? Now, imagine that you had to balance the weight on the tips of your fingers- it would feel a lot worse right? The force is the same- but in the second scenario, the force is acting over a much smaller area. You felt more pressure on your fingertips. If you’ve ever worn high heels, you are familiar with this concept. When you walk across a field in flats or barefoot, the earth holds you up just fine. If you put on heels, you’ll often find yourself sinking in: less area, greater pressure.

Think About It

You squeeze a tack between your index finger and your thumb. The flat part is against your finger, the pointer part is against your thumb. Which experiences more force- your finger or your thumb? Which experiences more pressure, your finger or your thumb? Which experiences more pain?

It is important to note that we only care about the force that acts perpendicular to the surface- we call this acting normal to the surface.

Because of this, you may sometimes see pressure written this way:

\[ \text{pressure} = \frac{\text{force}}{\text{area}} \]

\[ p = \frac{F}{A} \]

That little extra symbol (the upside down T) means normal to the surface.

Let’s talk about the units for pressure. Looking at the equation, you would expect the units to be N/m² and that would be accurate. We have a shorthand way of writing this in physics, which we call the Pascal.

\[ 1 \text{ Pascal (P)} = \frac{1 \text{ Newton (N)}}{1 \text{ m}^2} \]

A Pascal is an incredibly small amount of pressure. A nickel resting on a table applies a pressure of 140 Pascals. Because the numbers we are working with when we talk about fluids tends to be so huge, we often use kilopascals (1kPa = 10³ Pascals) or megapascals (1 MPa = 10⁶ Pa) (from Princeton Review, 401).
Atmospheres

There is another unit we often use when dealing with pressure— the Atmosphere. Believe it or not, you live at the bottom of a great sea of fluid, the same way a fish does. The only difference is your fluid is air. And the same way the weight of the water pushes down on a fish at the bottom of the ocean, the weight of the air pushes down on you. Not just you—it pushes down on the table in front of you, on the floor, on your backpack. It creates a standard pressure here at the bottom of the air sea. How much? 101,300 Pa. It’s a decent amount. Because this pressure is caused by our atmosphere, scientist will sometimes describe pressure using atmospheres.

\[ 101,300 \text{ Pa} = 1 \text{ atmosphere} \]

This will be given to you on the AP test, but it is still worth memorizing, as it takes precious time to look it up. Most people have it memorized by the end of this unit anyway, because they’ve used it so often.

Hydrostatic Pressure

When scientists visit the Titanic, they have to do so in a carefully pressured submarine. If they didn’t, the weight of the water would crush them like a tiny human grape. What causes this pressure? The weight of the water pushing down. We call this hydrostatic pressure (“hydro” = water, “static” = at rest) Imagine you laid a rug at the bottom of the sea, 100 m down. How much pressure would the rug feel due to the weight of the water above it?

Imagine the area above the rug as a column of water 100 m tall. Let’s figure out how much pressure this would cause on the rug.

\[
\text{Pressure} = \frac{\text{Force}}{\text{Area}}
\]

The force is caused by the weight of the water. Remember from last year that weight = \( F = mg \)

\[
\text{Pressure} = \frac{mg}{A}
\]
What is the mass of a column this size? Well, using $\rho = m/V$, we can rearrange so $\rho V = m$ and say:

$$P = \frac{\rho V g}{A}$$

The volume is the volume of the column of water above the rug. How could we calculate this? $V=Ah$. Let’s plug that in:

$$P = \frac{\rho (Ah) g}{A}$$

The area in the numerator and the area in the denominator cancel out leaving us with:

$$P = \rho gh$$

This is a key equation in fluids— and we can use it for a wide range of things— ships at the bottom of the ocean, calculating air pressure....

Practice Problem

What is the hydrostatic pressure on an object submerged in water at a depth of 15 m at or near sea level? ($\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$)

Solution

Using $P = \rho gh$, $P = (1.00 \times 10^3 \text{ kg/m}^3) (9.8 \text{ m/s}^2)(15 \text{ m}) = 147 \times 10^3 \text{ N/m}^2 = 1.47 \times 10^6 \text{ Pa}$

The amount you calculated above is the amount of pressure caused by the weight of the water. Above the surface of the water, however, there is air above that— an entire atmosphere also pushing down. In some situations, we also need to take that into account and add that on.

Practice Problem

What is the total pressure on the submerged object in the preceding example? Total pressure is often called absolute pressure.

Solution

The total pressure will equal the hydrostatic pressure of the water plus the atmospheric pressure:

$$P_{\text{total}} = P_{\text{hydrostatic}} + P_{\text{atm}}$$

$$= \rho gh + P_{\text{atm}}$$

$$= 1.47 \times 10^6 \text{ Pa} + 1.01 \times 10^5 \text{ Pa}$$

$$= 2.48 \times 10^6 \text{ Pa}$$

We have names for these two different pressures. If we are just talking about the pressure caused by the fluid (hydrostatic pressure), we call that gauge pressure. If we are talking about the pressure caused by the weight of the water and the weight of the atmosphere, we call the absolute pressure (total pressure).

$$P_{\text{gauge}} = \rho gh$$

$$P_{\text{absolute}} = P_{\text{gauge}} + P_{\text{atmosphere}}$$

On earth, this means:

$$P_{\text{absolute}} = \rho gh + 101 \text{ kPa}$$
Why is it called gauge pressure? Because gauges made in stores don’t take atmospheric pressure into account. If you are going to check your tires, you take out a gauge. Before you hook it up to the tire, the gauge reads zero—it doesn’t read “101,300 Pa”. The gauge ignores the atmospheric pressure, hence the name gauge pressure.

One important thing to note: all of these calculations would need to be carried out in Pascals in order for the equations to work. If you are given a pressure in atmospheres, always always convert to Pascals before moving on. Several students lost serious points on their end of the year test because they forgot to do this, and you will take a major point hit if you do it on the AP test.

Practice Problem

**Example 18.4** What is the hydrostatic gauge pressure at a point 10 m below the surface of the ocean? (Note: The specific gravity of seawater is 1.025.)

Solution

Solution. Using \( \rho_{\text{seawater}} = \text{sp. gr. } \times \rho_{\text{water}} = 1025 \text{ kg/m}^3 \), we find that

\[
p_{\text{gauge}} = \rho g h = (1025 \text{ kg/m}^3)(10 \text{ N/kg})(10 \text{ m}) = 1.025 \times 10^5 \text{ Pa}
\]

Practice Problem

**Example 18.5** A swimming pool has a depth of 4 m. What is the hydrostatic gauge pressure at a point 1 m below the surface?

Solution

Solution. Using \( \rho = \rho_{\text{water}} = 1000 \text{ kg/m}^3 \) for the density of the pool water, we find that

\[
p_{\text{gauge}} = \rho g h = (1000 \text{ kg/m}^3)(10 \text{ N/kg})(1 \text{ m}) = 1 \times 10^4 \text{ Pa}
\]

Notice that the total depth of the swimming pool is irrelevant; all that matters is the depth below the surface.

Practice Problem

**Example 18.6** What happens to the gauge pressure if we double our depth below the surface of a liquid? What happens to the total pressure?

Solution

Solution. Since \( p_{\text{gauge}} = \rho g h \), we see that \( p_{\text{gauge}} \) is proportional to the depth, \( h \). Therefore, if we double \( h \), then \( p_{\text{gauge}} \) will double. However, the total pressure, \( p_{\text{ atm}} + \rho g h \), is not proportional to \( h \). The term \( \rho g h \) will double, but the term \( p_{\text{ atm}} \), being a constant, will not. So, the total pressure will increase, but it will not double; in fact, it will increase by less than a factor of 2.
Level 2: Measuring Pressure

When we are talking about hydrostatic pressure, there are a few things you should keep in mind:

1. **Fluid pressure is exerted in all directions.** If you hold a box in your hand, the pressure the box exerts is only on the top of your hand. However, if you put your hand in water, the pressure exerted occurs to the entire surface of your hand. It pushes in on your hand from all directions.

2. **At any given depth, the pressure is the same.** If we go down to a particular depth and the pressure is 200 kPa, it will be the same anywhere else we go in that liquid as long as we stay at the same depth.

3. **The force due to a liquid is always perpendicular to a surface.** No matter how an object is shaped, the force caused by the hydrostatic pressure will always push in on an object perpendicularly. Right now, the weight of the air above you isn’t just pushing straight down on you (see Rule 1 in this list), is pushing in on your arms, in on the underside of your chin, in on the bottoms of your feet. It is acting perpendicularly to the curved surface of your skin.

These rules are really fun to play around with, and we are going to do that in a second. For now, let’s watch Ms. Twu. She’s going to help you clarify these ideas and will review some of the topics we have already discussed.
Twu Fluid Mechanics Video 3: Atmospheric Pressure, Absolute Pressure, and Gauge Pressure

Watch this video. You will often be watching videos in this class. Most groups choose to gather around one device and watch the video together. When someone is confused, they can stop the video and discuss it as a group. Some groups prefer to put headphones on and watch it individually, then talk about it afterward. Do whatever works for you- just make sure you are talking to one another and pausing the video when you are confused.

http://youtu.be/-2SFyb4K7r8

Barometers

Scientists are some pretty crafty people. They looked at the three rules above and realized there was a lot they could do with it. For example, every day, the pressure of the atmosphere changes a little bit (the pressure I gave you before was an average). You can see this on the weather channel- when they talk about barometric pressure, that’s what they mean. Barometric pressure (atmospheric pressure) often changes just before storms, so if you can read changes in pressure, you can start to predict the weather. You can even do this at home, pretty easily.

If we want to track changes in atmospheric pressure, we are going to need to be able to measure atmospheric pressure. To do this, scientists came up with a slick little device called a barometer. Here is:

![Barometer Diagram](image)

To create a barometer, you need a tall column of a fluid that has a vacuum (a space without air/gas) at the top. There needs to be a vacuum there because we don’t want the pressure of the gas pushing down on the column of liquid and messing up our values.

The column of liquid is then set up in a small pool that is open to the atmosphere.

Let’s look at two points in this set up- I’ve marked them with A and B. Point A is at the surface of the liquid. Point B is at the very base of the column of liquid. Once the barometer is set up and still, we know that the pressure at Point A and Point B need to be the same- if they weren’t, the fluid would flow, trying to reach equilibrium.

\[ P_A = P_B \]

The pressure at Point A is caused by the atmosphere pushing down on the liquid. The pressure at Point B is caused by the pressure of the column of liquid pushing down on it. Therefore, we can say:

\[ P_A = P_B \]
\[ P_{atm} = \rho_f g h \]

where \( \rho_f \) is the density of the fluid, g is acceleration due to gravity, and h is the height of the column. Here’s where it gets slick. Look closely at the equation. The density of the fluid and g (9.8) won’t change. This means that if the atmospheric pressure changes, the height of the column will have to change.
**Practice Problem**

Let’s say the atmospheric pressure outside was 102 kPa and you were in the mood to make your own barometer. You set up the pan, and get some water for the column. At this pressure, how tall would the column of water be?

**Solution**

In order to solve this problem, you had to remember that the density of water is 1000 kg/m$^3$. Again, if you haven’t memorized this yet, you might want to now, because it makes problem solving a lot easier.

\[ P_{\text{atm}} = \rho_f gh \]

Remember to solve for the variable you are interested in first.

\[ P_{\text{atm}} = h \]

Now you can plug number in... and be careful to actually show yourself plugging the numbers in... you often receive points for this on the AP test.

\[ \frac{102 \times 10^3}{(1000)(9.8)} = h \]

\[ h = 10.4 \text{ m} \]

Ugh... that’s not going to fit in your house, unless you are way wealthier than I am, in which case hook me with some lab supplies.

Look at the equation again. What could we do to make a barometer that would actually fit in a classroom?

If you answered that we could get a more dense liquid, you, sir, are on your way to great things.

This is why barometers are often made of mercury. Water has a density of 1000 kg/m$^3$- mercury has a density of 13600 kg/m$^3$. You don’t have to memorize this. If it is on the test, they will give it to you.

**Practice Problem**

At standard pressure at sea level, how tall would a column of mercury be in a barometer?

**Solution**

Standard pressure was (again, memorize this) 101300 Pa. Plugging that in, along with the new density of the fluid:

\[ P_{\text{atm}} = \rho_f gh \]

\[ P_{\text{atm}} = h \]

\[ \frac{101300}{(13600)(9.8)} = h \]

\[ h = 0.760 \text{ m} \]

This is the way chemists like to talk about atmosphere- as 760 mm Hg, in other words 760 millimeters of mercury. You may have heard this before- now you know where it comes from. They are talking about the height of a column of mercury.

**Twu Fluids Video 7: U-Shape Tube** Ms. Twu will show you another variation on this.

http://youtu.be/qG6cfwQBhHM
U Bend Problems

Another interesting problem we can solve using these ideas is the U-Bend Problem. Imagine that you had a tube of plastic that you bent into a u shape, with both open ends pointing up toward the ceiling. You pour water in. You would expect that the water would level out, and you’d have water on both sides, like this:

What do you think would happen if you then poured an oil into one side? Let’s say you poured in vegetable oil, which has a density of 920 kg/m$^3$ (a little lower than water). Would the liquids on either side even out again, or would one be higher?

One side would be higher - the side you added the oil to. Why? Let’s choose two points near the bottom of each column of fluid to figure out why.

Let’s look at the two columns. Point A is at the very bottom of the column of oil. Column B is straight across the way, at the same level. Point A has oil above it, fluid below. Column B has water above it, water below. One thing we can say for certain is that Point A and Point B have the same pressure. If they didn’t the fluid would try to flow toward the area of lower pressure. Since the fluid is still, we know it has reached equilibrium and the pressures are equal.

$$P_A = P_B$$

What causes the pressure on Point A? The column of oil above it.

$$P_A = \rho_o g h_o$$

What causes the pressure on Point B? The column of oil above it

$$P_B = \rho_w g h_w$$

Set them equal to each other:

$$P_A = P_B$$

$$\rho_o g h_o = \rho_w g h_w$$

The “g”s cancel, leaving us with:

$$\rho_o h_o = \rho_w h_w$$

Why does the oil (with a density of 920 kg/m$^3$) have a taller column than the water side (with density 1000 kg/m$^3$)? Take a look at the equation. In fact, take this chance to calculate the height of the column of water above point B. Let’s say the height of the oil column is 11 cm.

$$\rho_o h_o = \rho_w h_w$$

$$(920)(0.11) = (1000)h_w$$

$$0.10 \text{ m} = h_w$$
If the height of the oil column was 11 cm and the height of the water column was 10 cm, they have a difference of 1 cm.

*Think About It*

If you wanted there to be a bigger difference in heights in the two columns, what could you do?

**Level 3: Pascal’s Principle**

Pascal’s Principle is one of the most useful ideas in fluids. Once you understand how it works, you will start to see tons of objects in your everyday world that rely on it. It’s actually a pretty simple idea. Here’s the official wording:

> Pressure is transmitted undiminished in an enclosed static fluid.

Okay, now in English: if you apply pressure to one part of a fluid, that pressure will transfer to the rest of the fluid. For example, in the picture, I have a syringe with a glass ball on the end. If I apply a pressure of 6000 Pa to the stopper, the pressure in the fluid (and on the walls of the fluid) will increase by 6000 Pa too. You understood this idea as a kid - if you squeeze one end of a long inflated balloon, you will see the other end expand to try to compensate for the increase in pressure.

This idea is incredibly useful. We use this idea to create hydraulic systems: hydraulic lifts, hydraulic brakes, hydraulic robotic arms... Here is how it works. Imagine you hooked two syringes up to one another using a long plastic tube, like so:

If you push down on the syringe on the bottom, you increase the pressure on the system - which is transmitted into the tube, into the next syringe. That syringe will then move. This way, you can easily transfer forces from one place to another.

**Basic Hydraulic Lift System:** Watch this video, which will show you how hydraulic lifts work. Try not to let the kid in the video make you feel stupid. Girl is wicked smart.

[http://www.youtube.com/watch?v=uy4t0ueWEso](http://www.youtube.com/watch?v=uy4t0ueWEso)

Once you see this one place - you will see it everywhere. Remember the brakes on your mountain bike? How do you think it worked when you squeezed the brake?

There is another component to this that makes it even more amazing. Imagine that instead of hooking up two syringes that were roughly the same size, we set up a system like this:
Notice that the two column on either sides have different areas. How does this effect anything? Well, remember that if we apply a change in pressure to the column on the right, we have to experience the same change in pressure in the column on the left so:

\[ \Delta P_1 = \Delta P_2 \]

and if you recall:

\[ P = \frac{F}{A} \]

If we apply a force to Column 1, how much force would act on Column 2?

\[ \Delta P_1 = \Delta P_2 \]

\[ \frac{F_1}{A_1} = \frac{F_2}{A_2} \]

This means that we can apply a little force to the left side and get a greater force out on the right side. This idea of putting a smaller force in and getting a larger force out is called mechanical advantage. You may have studied it in your previous physics class. How is this possible? It doesn’t break the law of conservation of energy because you would need to push the guy on the left down further. Still pretty neat system.

This is how your brakes work:
Practice Problem: Hydraulic Jack and a Person

Consider the hydraulic jack diagramed below. A person stands on a piston that pushes down on a thin cylinder full of water. This cylinder is connected via pipes to a wide platform on top of which rests a 1-ton (1000 kg) car. The area of the platform under the car is 25 m²; the person stands on a 0.3 m² piston. What is the lightest weight of a person who could successfully lift the car?

Solution

We have an enclosed fluid, so Pascal's principle insists that the pressure applied by the person should also be applied equally to the car. Well, we know what pressure will be necessary to lift the car: Pressure is force/area, or the weight of the car over the area of the car's platform:

\[ P = \frac{10,000 \text{ N}}{25 \text{ m}^2} = 400 \text{ Pa} \] (recall that a pascal, Pa, is a newton per m²).

So the person needs to provide an equal pressure. The person's weight is applied over a much smaller area, only 0.3 m². So set 400 Pa equal to the person's weight divided by 0.3 m².

Practice Problem: Hydraulic Lift and Mass
A hydraulic lift consists of a platform that is supported by a cylindrical piston, which resides in a tube connected to a reservoir of hydraulic fluid (figure 4). The piston and platform have a mass of 300 kg. The input force is generated at a smaller piston connected to the same reservoir. If the radius of the smaller piston is 2 cm, what must be the radius of the larger piston so that a moderate force of 100 N will lift a 1,700 kg car?

**Solution**

The input force must support the car, piston, and platform with total mass 2,000 kg. From the preceding formula you can write:

\[
\frac{A'}{A} = \frac{F'}{F} = \frac{2,000(10)}{100} = 200
\]

\[
\frac{\pi r^2}{\pi (2)^2} = 200 \Rightarrow r = 28.3 \text{ cm}
\]

**Practice Problem: Hydraulic Problems with Variables**

A hydraulic press has one piston of radius \( R_1 \) and another piston of radius \( R_2 \). If a 400.0-N force is applied to the piston of radius \( R_1 \) and the resulting force exerted on the other piston is 1600.0 N, which of the following is a correct mathematical statement concerning each piston radius?

(A) \( R_2 = R_1 \)

(B) \( R_1 = 2R_2 \)

(C) \( R_1 = \sqrt{2} \ R_2 \)

(D) \( R_2 = 2R_1 \)

(E) \( R_2 = 2\sqrt{2} \ R_2 \)

**Solution**

The pressure is uniform within the press, so, by Pascal's principle, \( F_1A_2 = F_2A_1 \), which contains each piston's value of \( R \) under area \( A \). The area is \( \pi r^2 \), which becomes \( F_1(\pi R_2^2) = F_2(\pi R_1^2) \). Solving for the ratio of the radii gives \( \frac{R_2^2}{R_1^2} = \frac{F_2}{F_1} \), which becomes \( R_2 / R_1 = \sqrt{1600/400} \). Substituting the values of the forces gives \( R_2 / R_1 = \sqrt{1600/400} \) or \( R_2 / R_1 = 2 \). Therefore, \( R_2 = 2R_1 \), which is choice (D) (College Physics 7th ed. pages 277-281/6th ed. pages 279-283).
Level 4: Buoyancy

Introduction to Buoyant Force

When we place an object into a fluid, it experiences a slight force pushing it up. You are already very aware of this, and you have been since you were a child— it’s why you liked to push beach balls under the water, why you liked to jump around in the shallow side of the pool, feeling light-weight, why heavier objects felt mysteriously a little lighter in water. All of this occurs due to **buoyancy**— an upward force acting on objects due to liquids.

Why does this happen? Why do objects feel an upward push at all? To understand this, try out the next problem. It might sound confusing at first, but I promise you that you already know the ideas behind this.

Practice Problem

A rectangular object is submerged in water (see picture). The object is 3 cm tall, and it is completely submerged 2 cm below the water. What is the difference in pressure between the top and bottom of the object?

**Solution**

The difference in pressure is given by \( p_1 - p_2 = \rho g h_1 - h_2 \).

\[
p_1 - p_2 = (1.00 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.05 \text{ m} - 0.03 \text{ m}) = 1.96 \times 10^2 \text{ Pa}
\]

Think about this problem: the bottom of the block (which is deeper in the water) is experiencing more pressure than the top of the block. This means, essentially, that the object is experiencing an overall force acting upward on it. This is the basic idea behind why objects experience a buoyant force.

Archimedes’ Principle

The above problem might demonstrate the key idea, but that was a pretty simple problem. What if we wanted to deal with much more irregular shapes, like jagged rocks, or swans, or boats? This calculation could get messy fast. Luckily, there is actually an easier way to think about/solve problems involving buoyant force. Buoyancy can be explained really well using **Archimedes’ Principle**. That dude in the pic is Archimedes. We’ll talk about why he’s naked and how he came up with this whole thing shortly, but that might confuse you, so let’s learn the principle first.

**Archimedes’ Principle:**

The buoyant force acting on an object is equal to the weight of the water displaced by the object.

Sounds confusing now, but it’s actually not that confusing. Let’s take it a step at a time.

Imagine you have a tennis ball, with a volume \( 1.1 \times 10^{-4} \text{ m}^3 \). For the sake of explaining this, let’s say the ball hasn’t started to bob up to the surface yet. It is completely under water. What would the buoyant force be on this ball?
According to Archimedes’ Principle, we need to figure out the weight of the water displaced by the tennis ball. Displaced is an educated word for “knocked out of the way.” When you put the tennis ball into the water, it pushed some of the water out of the way in order to make space for itself. If we can find the weight of the amount of water it pushed out of the way (displaced), we would know the buoyant force acting on the tennis ball. One important thing to remember is that the volume of water pushed out of the way is the same as the volume of the ball.

\[ V_{ball} = V_{displaced \text{ water}} \]

That’s great, but we want the weight of the displaced water. In order to do that, we are going to know the mass of the water. An equation that can help us with this is:

\[ \rho = \frac{m}{V} \]

\[ \rho V = m \]

We know the volume of the displaced water (same as the volume of the water) and we know the density of water because we’ve already memorized it (right? right?!)

\[ m = p_{\text{water}} V_{\text{water}} \]

That tells us the mass. How do we figure out the weight? Get ready for a blast from the past:

\[ F_g = mg \]

Weight= force of gravity! So...

\[ F_g = \rho_{\text{water}} V_{\text{water}} g \]

Archimedes’ Principle says that the weight of the water displaced is equal to the buoyant force.

\[ F_{\text{Buoyant}} = F_g = \rho_{\text{water}} V_{\text{water}} g \]

We can actually make this a pretty easy equation. One thing: we don’t always know that we are going to be submerging the object in water- we might want to submerge it in oil or mercury or spaghetti sauce… A more general version of this equation is:

\[ F_{\text{Buoyant}} = \rho_{\text{liquid}} V_{\text{liquid}} g \]

Because \[ V_{\text{liquid}} = V_{\text{object submerged}} \] I like to think of this equation this way:

\[ F_{\text{Buoyant}} = \rho_{\text{liquid}} g V_{\text{submerged}} \]

And this is the mathematical version of Archimedes’ Principle. You need to memorize this.

Now, the tennis ball problem is easy. We said before that the tennis ball has a volume of \(1.1 \times 10^{-4}\). Let’s plug n’ chug.

\[ F_{\text{Buoyant}} = \rho_{\text{liquid}} g V_{\text{submerged}} \]

\[ F_{\text{Buoyant}} = (10^3)(9.8)(1.1 \times 10^{-4}) \]

\[ F_{\text{Buoyant}} = 1.078 \text{ N} \]
Does this mean the tennis ball would float or sink? In order to figure out that, we need to think about our old friend the free body diagram. Draw a free body diagram of the tennis ball submerged under water.

We can use this to find the net force on the object

\[ F_{\text{net}} = F_B - F_g \]
\[ F_{\text{net}} = F_B - mg \]

A tennis ball has a mass of 0.057 kg

\[ F_{\text{net}} = 1.1 N - (0.057)(9.8) \]
\[ F_{\text{net}} = 0.54 N \]

This is a positive number, meaning the buoyant force is “winning”. The tennis ball would (of course) go up toward the surface. **Note: the picture isn’t very good. Buoyancy should be longer.**

**Video Demonstration:** In this video, Paul Hewitt does a great job demonstrating the idea behind Archimedes’ Principle.

[http://www.youtube.com/watch?v=g6aErhwFXsg](http://www.youtube.com/watch?v=g6aErhwFXsg)

**Practice Problem**

What is the buoyant force exerted on a ball of diameter 0.2 m if immersed in seawater at or near sea level? (\( \rho_{\text{seawater}} = 1.03 \times 10^3 \text{ kg/m}^3 \))

**Solution**

Using \( F_{\text{buoy}} = \rho V g \)

\[ F_{\text{buoy}} = \left( \frac{4\pi}{3} \right)(0.1 \text{ m})^3(1.03 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2) = 42.3 \text{ N} \]

**Practice Problem**

If the ball in the preceding example has a mass of 0.5 kg, does it sink?

**Solution**

The ball will sink if its weight is greater than the buoyant force. Its weight is \( F_{\text{wt}} = (mg) = (0.5 \text{ kg})(9.8 \text{ m/s}^2) = 4.9 \text{ N} \), which is less than the buoyant force of 42.3 N. Therefore, the ball will float.

On the surface, these seem pretty easy. But buoyancy problems are notorious for messing with your head. Try this one:

**Practice Problem**

You have two blocks- each with a length of 1 cm, a width of 1 cm, and a height of 3 cm. One block is made of chromium (density=7.1 x 10^3 kg/m^3), the other is made of oak (0.72 x 10^3 kg/m^3). Which one would experience the greater buoyant force, if they were submerged in water? Do both float? Does one sink?
Solution

Here’s the part that will mess with your head- I can tell you without touching my calculator, without writing anything on paper, that both of these objects have the same buoyant force. How did I know that? Because both of these objects have the same volume, which means they will displace the same amount of water, which means the weight of the water they displaced has to be equal, which means the buoyant force acting on both of them has to be equal. Trick question!

I’ll tell you right now that the chromium is going to sink and the oak is going to float. How is that possible if they both have the same buoyant force acting on them? Don’t forget their weight. Sure- the chromium will have the same buoyant force acting on it, but it also has more weight- it is heavier. For the chromium, the buoyant force isn’t enough to counteract its weight. The oak is made of a lighter substance, so the buoyant force is greater than the weight- it will bob up toward the surface. Let’s prove this to ourselves mathematically.

First, let’s find the buoyant force acting on these guys.

\[ F_B = \rho_{\text{fluid}} V_{\text{submerged}} g \]

We have the density of the fluid (water, 10³). We have \( g \) (9.8). We need the volume.

\[ V = \text{width} \times \text{height} \times \text{length} = \text{whl} \]

So the volume of both of the boxes is:

\[ V = (0.01) \times (0.03) \times (0.01) = 3 \times 10^{-6} \]

Plug that back in

\[ F_B = \rho_{\text{fluid}} V g \]

We know (using a free body diagram) that the net force will be:

\[ F_{\text{net}} = F_B - F_g \]

So...

\[ F_{\text{net}} = \rho_{\text{fluid}} V g - F_g \]

\( F_g = m_{\text{box}} g \) so we can say:

\[ F_{\text{net}} = \rho_{\text{fluid}} V g - m_{\text{box}} g \]

They didn’t give us the mass of the boxes, though, only the density. Never fear. We know that the density and mass are related, and we know the volume, so we can say: \( \rho = \frac{m}{V} \), and therefore \( m = \rho V \)

\[ F_{\text{net}} = \rho_{\text{fluid}} V g - \rho_{\text{box}} V g \]

Both boxes are submerged in the same liquid, so they will have the same buoyancy. They have different weights, however. We can plug n’ chug at this point to figure out their net force.
Oak

\[ F_{\text{net}} = \rho_{\text{fluid}} V g - \rho_{\text{oak}} V g \]

\[ F_{\text{net}} = (3 \times 10^{-6})(9.8) - (0.72 \times 10^{3})(3 \times 10^{-6})(9.8) \]

\[ F_{\text{net}} = 0.0294 - 0.0211 \]

\[ F_{\text{net}} = 0.0083 \text{ N} \]

\[ F_{\text{net.oak}} = 0.0083 \text{ N} \]

The positive value means the buoyant force “won” and the oak moved up.

Chromium

\[ F_{\text{net}} = \rho_{\text{fluid}} V g - \rho_{\text{oak}} V g \]

\[ F_{\text{net}} = (10^{3})(3 \times 10^{-6})(9.8) - (7.1 \times 10^{3})(3 \times 10^{-6})(9.8) \]

\[ F_{\text{net}} = 0.0294 - 0.209 \]

\[ F_{\text{net}} = -0.18 \text{ N} \]

\[ F_{\text{net.chromium}} = -0.18 \text{ N} \]

The negative sign means that the weight is greater than the buoyant force, so the box sinks.

**Video Animation:** Watch this short TED animation to understand where Archimedes’ Principle came from, and why it is often associated with a naked dude.

[http://www.youtube.com/watch?v=ijj58xD5fD1](http://www.youtube.com/watch?v=ijj58xD5fD1)

**Level 5: Percent Submerged and Apparent Weight**

In the last section, we talked about why objects float (or sink). In this section, we are going to look at even more applications of Archimedes Principle. Let’s start by taking a closer look at objects that are floating on the surface of the fluid.

It is important to realize that for floating objects, they are no longer completely submerged. Part of their volume is above the fluid, and part is below. How much is above and how much is below can vary. Think of a partially filled water bottle versus a beach ball. You would expect more of the water bottle to be submerged, while the beach ball would only have a small percentage underwater, so that it would seem to float on the surface.

We can quite easily calculate how much of an object will be submerged. We need to start by realizing that for floating objects, the weight and the buoyant force are equal. They have to be - if the buoyant force was greater, the object would be rocketing out of the water. If the weight was greater, the object would sink. So we can say:

\[ F_B = F_g \]

Remember that \( F_B = \rho_{\text{fluid}} V_{\text{submerged}} g \) and \( F_g = mg \)

\[ F_B = F_g \]

\[ \rho_{\text{fluid}} V_{\text{submerged}} g = mg \]

See anything that can cancel?

\[ \rho_{\text{fluid}} V_{\text{submerged}} = m \]
Let’s set that mass in terms of density and volume. \( \rho = \frac{m}{V} \) \( m = \rho_{\text{object}} V_{\text{object}} \) Note that the volume here is the total volume of the object. We aren’t just talking about the part that is under the water- we’re talking about the whole object here.

\[
\rho_{\text{fluid}} V_{\text{submerged}} = \rho_{\text{object}} V_{\text{object}}
\]

If we read this equation in English, it is saying:

(density of the fluid)*(volume of the object that is in the fluid) = (density of the object)*(total volume of the object)

We often rearrange this into an easier to memorize (and use) format:

\[
\frac{\rho_{\text{fluid}}}{\rho_{\text{object}}} = \frac{V_{\text{object}}}{V_{\text{submerged}}}
\]

Let’s use this to solve a practice problem, and the we’ll come back and talk more about what it means.

**Practice Problem**

Ice has a specific gravity of 0.91. Imagine you have a block of ice that is 0.2 \( m^3 \). If you were to put this block of ice in Freon, which has a specific gravity of 1.49, what would the volume of the exposed portion of the ice be?

**Solution**

\[
\frac{\rho_{\text{fluid}}}{\rho_{\text{object}}} = \frac{V_{\text{object}}}{V_{\text{submerged}}}
\]

\[
V_{\text{submerged}} = \frac{V_{\text{object}} \rho_{\text{object}}}{\rho_{\text{fluid}}}
\]

\[
\rho = (\text{specific gravity})(\rho_{\text{water}})
\]

\[
\rho_{\text{ice}} = (s.g, \text{ice})(\rho_{\text{water}})
\]

\[
\rho_{\text{freon}} = (s.g, \text{freon})(\rho_{\text{water}})
\]

\[
V_{\text{submerged}} = \frac{V_{\text{ice}} \rho_{\text{ice}}}{\rho_{\text{freon}}}
\]

\[
V_{\text{submerged}} = \frac{V_{\text{ice}} (s.g, \text{ice})(\rho_{\text{water}})}{(s.g, \text{freon})(\rho_{\text{water}})}
\]

\[
V_{\text{submerged}} = \frac{V_{\text{ice}} (s.g, \text{ice})}{(s.g, \text{freon})}
\]

\[
V_{\text{submerged}} = \frac{(0.2)(910)}{(1490)}
\]

\[
V_{\text{submerged}} = 0.122 \ m^3
\]

This is a little bit tricky. We just did all that work to calculate the amount under the water. But the problem asked us for the volume above the water. Therefore, we need to say:

\[
V_{\text{total}} - V_{\text{submerged}} = V_{\text{exposed}}
\]

\[
0.2 - 0.122 = V_{\text{exposed}}
\]

\[
0.078 \ m^3 = V_{\text{exposed}}
\]
Note that halfway through the previous problem, we sort of derived another useful equation (I highlighted where this happened for you):

\[
\frac{V_{\text{submerged}}}{V_{\text{object}}} = \frac{(s.g, \text{object})}{(s.g, \text{fluid})}
\]

It is perfectly valid to use this equation whenever you are given specific gravity instead of density.

**Percent Submerged**

These type of problems will very rarely ask us to find the volume, however. They normally like to ask for either the fraction of the object exposed/submerged, or they like to ask for the percent of the object exposed/submerged. This sounds more complicated, but it is actually quite a bit easier that the problem we just solved.

You often see pictures of icebergs like this:

Is this realistic? Do icebergs really have that much of their body hidden below the water? Let’s find out.

**Practice Problem**

An iceberg has a specific density of 0.920. Seawater has a specific gravity of 1.025. What percent of an iceberg can you see above the water?

**Solution**

\[
\frac{V_{\text{submerged}}}{V_{\text{object}}} = \frac{(s.g, \text{object})}{(s.g, \text{fluid})}
\]

\[
\frac{V_{\text{submerged}}}{V_{\text{object}}} = \frac{0.920}{1.025}
\]

\[
\frac{V_{\text{submerged}}}{V_{\text{object}}} = 0.898
\]

That means that 89.8% of an iceberg is submerged. 100-89.8= 10.2% of the iceberg is exposed. Does that make this picture accurate?

**Practice Problem: Solving for Amount Submerged**

Example 18.9 An object with a mass of 150 kg and a volume of 0.75 m³ is floating in ethyl alcohol, whose specific gravity is 0.8. What fraction of the object’s volume is above the surface of the fluid?
Solution

The density of the object is

\[ \rho_{\text{object}} = \frac{m}{V} = \frac{150 \text{ kg}}{0.75 \text{ m}^3} = 200 \text{ kg/m}^3 \]

Because the density of the fluid is \( \rho_{\text{water}} = 1000 \text{ kg/m}^3 \), the ratio of the object's density to the fluid's density is

\[ \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \frac{200 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = \frac{1}{4} \]

This means that 1/4 of the object's volume is below the surface of the fluid; therefore, the fraction that's above the surface is 1 - (1/4) = 3/4.

Apparent Weight

When objects are submerged in fluid, they seem to weigh a little less. We explored why this happens in the previous buoyance section. How much an object seems to weigh in the fluid is called the apparent weight, which we show using the symbol \( F' \). We've already dealt with this idea in previous problems, we just didn't put a term to it.

Let's say you have a mass of 2 kg. Obviously, if it is resting on a table, it has a weight of 19.6 N \( (F_g = mg) \). If you were to hang the mass on a spring scale, it should read 19.6 N. Now, imagine dipping that mass in a tub of water. The reading changes... it now says that the mass has a weight of 14 N. What does this mean?

In the first situation, where you just had the mass hanging on the end of a spring scale, the tension of the spring scale pulling up is the same as the force of the weight pulling down. When you put the mass into water, however, you now have the buoyant force pushing up. This will make the reading on the scale decrease. The amount the weight seems to change, therefore, is your buoyant force.

\[ F_g - F'_g = F_B \]
Occasionally, completely vile people will talk about the apparent mass, whose symbol is \( m' \). This is just how much mass the object seems to have, if you were to trust the value of the apparent weight.

\[ F'_g = m'g \]

I hate this concept, because it makes it seem like mass changes when you submerge it in liquid, which, of course, it most certainly does not. But you can see it every now and then. Here’s a practice problem.

**Practice Problem**

1. When submerged under water, the apparent mass of one cubic meter of pure gold is 18300 kg. What would be its mass in air?
   
   (A) 16300 kg  B) 17300 kg  C) 18300 kg  D) 19300 kg  E) 20300 kg

**Solution**

1 m\(^3\) volume cube under water displaces 1m\(^3\) of water. This weight of water = \( pVg = 1000(1)(10) = 10000 \text{ N} \) which is equivalent to the buoyant force. The apparent weight in water is \( mappg = 18300(10) = 183000 \text{ N} \). This apparent weight is lessened by the buoyant force pulling up with 10000 N of force. So outside of the water, this upwards force would not exist and the actual weight would be 193000 N which equal 19300 kg of mass.